FRICTIONAL DRAG WITH LONGITUDINAL RIBBING OF SURFACE IN FLOW

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An approximate theoretical scheme is proposed for calculating the turbulent boundary layer at a surface with longitudinal ribs of triangular cross section.

Recent experiments [1-4] have shown the possibility of a 7-10% reduction (in certain conditions) in the frictional drag at an incompressible turbulent boundary layer developing along a surface with longitudinal ribs of triangular cross section. An approximate calculation method is outlined below for estimating the influence of longitudinal ribs of triangular cross section on the drag at the surface with turbulent flow conditions in the boundary layer of an incompressible liquid. The method is based on the conventional assumptions of the semiempirical theory of a turbulent boundary layer (TBL). No additional experimental dependences are required to obtain the theoretical results, which are compared with known experimental data.

1. Two factors are taken into account in flow around a surface with longitudinal ribs whose height is small in comparison with the TBL thickness:

the increment in wet surface, which leads to increase in drag, other conditions being equal;

the appearance of additional solid impermeable surface elements in the wall region of the turbulent flow; in view of the impermeability and adhesion conditions, the mean and pulsational components of the liquid-particle velocity vector are zero at these elements; the appearance of regions close to the wall which have a lower level of velocity pulsations than a smooth surface results in decrease in the turbulent mixing of the flow, which may lead to reduction in drag in the TBL.

To estimate the total effect of flow around a ribbed surface, triangular ribs are investigated (Fig. 1).

Consider longitudinal flow around triangular ribs. Experiments indicate that reduction in drag is observed when the rib height is commensurate with the thickness of the viscous (laminar) sublayer of the TBL. Therefore, the flow in the space between the ribs is assumed to be laminar. Suppose that the mean-velocity gradients normal to the rib surfaces are considerably greater than the longitudinal gradient. The change in flow rate in the boundary layer along the surface is taken into account by a quasi-local approach widely used in TBL theory to describe liquid motion in the wall zone [5]. In the coordinate system (r, z, θ) in Fig. 1, this condition is equivalent to the following assumption regarding the velocity components

$$v_z = v(r, \theta);\ v_r = v_\theta = 0;\ \frac{\partial p}{\partial r} = \frac{\partial p}{\partial z} = \frac{\partial p}{\partial \theta} = 0, \tag{1}$$

where p is the pressure in the space between the ribs.

Taking account of Eq. (1), the continuity equation is satisfied identically, and the system of Navier–Stokes equations reduces to a single equation

$$\frac{\partial^2 v}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 v}{\partial \theta^2} + \frac{1}{r}\frac{\partial v}{\partial r} = 0. \tag{2}$$

The stress in the liquid is determined by the relations

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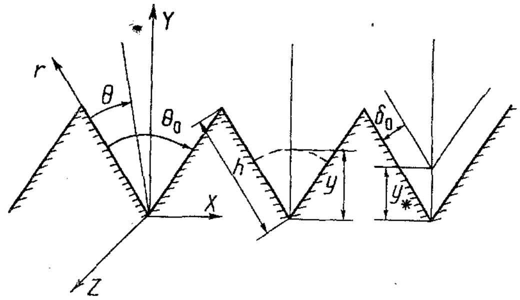

Fig. 1. Rectangular [X, Y, Z] and cylindrical [r, θ, Z] coordinate systems associated with ribbed surface.

$$\tau_{\theta z} = \mu \frac{1}{r}\frac{\partial v}{\partial \theta};\ \tau_{zr} = \mu \frac{\partial v}{\partial r}. \tag{3}$$

Taking account of Eq. (3), the frictional force within the dihedral angle is determined by the integral

$$F = 2\mu \int_0^h \frac{1}{r}\frac{\partial v}{\partial \theta}\bigg|_{\theta=0} dr, \tag{4}$$

where h is the length of a rib of triangular cross section.

The boundary conditions for v(r, θ) are found from the following considerations:

a) from the adhesion condition at the walls

$$v(r,\ 0) = v(r,\ \theta_0) = 0; \tag{5}$$

$$v(0,\ \theta) = 0; \tag{6}$$

b) since the liquid between the ribs moves on account of the tangential stress transmitted by the basic flow, the longitudinal velocity at r = h must be specified

$$v(h,\ \theta) = A \sin \frac{\pi\theta}{\theta_0}. \tag{7}$$

The form of the right-hand side of Eq. (7) is determined by the liquid adhesion at the faces $[v(h,\ \theta_0) = v(h,\ 0) = 0]$ and the smoothness of velocity variation over the angle θ. The constant A is found from the condition

$$\int_0^{\theta_0} v(h,\ \theta)\, d\theta = \int_0^{\theta_0} \frac{du}{dy} h \cos\left(\frac{\theta_0}{2} - \theta\right) d\theta, \tag{8}$$

where du/dy is the mean velocity gradient, which remains to be determined. The condition in Eq. (8) is invalid when $\theta_0 \to 0$; therefore, the limiting case $\theta_0 \to 0$ cannot be considered in the subsequent formulas containing A. Equation (8) expresses the equality of the mean longitudinal velocities of two flows in the region of the upper boundary of the region occupied by the ribs: the flow from the real ribs and some conventional turbulent flow at a wall with distributed equivalent roughness. Substitution of Eq. (7) into the left-hand side of Eq. (8) gives

$$A = \pi h \frac{du}{dy} \frac{\sin \theta_0/2}{\theta_0}.$$

The boundary condition is written in the form

$$v(h,\ \theta) = \pi h \frac{du}{dy} \frac{\sin \theta_0/2}{\theta} \sin \frac{\pi\theta}{\theta_0}. \tag{9}$$

The general solution of Eq. (2) is sought in the form

$$v(r,\ \theta) = R(r)\Phi(\theta),$$

which leads to the following relations on taking account of the boundary condition in Eq. (5)

$$\Phi'' + k^2\Phi = 0, \tag{10}$$

$$r\frac{d}{dr}\left(r\frac{dR}{dr}\right) - k^2R = 0. \tag{11}$$

The general solution of Eq. (10) takes the form

$$\Phi(\theta) = C_1 \sin k\theta + C_2 \cos k\theta.$$

Finding $C_2 = 0$, $\sin k\theta_0 = 0$ from Eq. (5), $k = \pi n/\theta_0$ may be determined, where $n = 1, 2, \ldots$. Hence

$$\Phi(\theta) = C_1 \sin\frac{n\pi\theta}{\theta_0}.$$

Seeking the solution of Eq. (11) in the form $R(r) = r^p$, it is found that $p = \pm k$, $R(r) = D_1 r^k + D_2 r^{-k}$.

It follows from the boundary condition in Eq. (6) that $D_2 = 0$. Thus, the particular solution of Eq. (2) satisfying the boundary conditions in Eqs. (5) and (6) takes the form

$$v_n(r, \theta) = B_n r^{\frac{\pi n}{\theta_0}} \sin\frac{\pi n\theta}{\theta_0}.$$

The general solution of the equation is

$$v(r, \theta) = \sum_{n=1}^{\infty} B_n r^{\frac{\pi n}{\theta_0}} \sin\frac{\pi n\theta}{\theta_0},$$

where the constants B_n are determined from Eq. (9)

$$B_1 = \pi\frac{du}{dy}\frac{\sin\theta_0/2}{\theta_0} h^{1-\frac{\pi}{\theta_0}};\quad B_2 = B_3 = \ldots = B_n = 0.$$

Thus, the solution of Eq. (2) satisfying Eqs. (5), (6), and (9) is obtained

$$v(r, \theta) = \pi\frac{\sin\theta_0/2}{\theta_0}\frac{du}{dy} h\left(\frac{r}{h}\right)^{\frac{\pi}{\theta_0}} \sin\frac{\pi\theta}{\theta_0}. \tag{12}$$

Using Eqs. (4) and (12), the friction on the section of the dihedral angle characterized by the coordinate $r = r_1$ is determined

$$F(r_1) = 2\mu\int_0^{r_1}\frac{1}{r}\frac{\partial v}{\partial\theta}\bigg|_{\theta=0} dr = 2\mu\left(\frac{\pi}{\theta_0}\right)\frac{du}{dy} h\left(\frac{r_1}{h}\right)^{\frac{\pi}{\theta_0}} \sin\theta_0/2. \tag{13}$$

2. The basic equation for the TBL velocity profile in the wall region expresses the condition of constant tangential stress in the direction transverse to the surface [5]

$$\tau_{\ell a} + \tau_T = \tau_w = \text{const}, \tag{14}$$

where $\tau_{\ell a}$ is the laminar friction; τ_T is the turbulent friction; τ_w is the total frictional stress, equal to the friction at the wall.

At a smooth surface

$$\tau_{\ell a0} = \mu\frac{du}{dy},$$

which corresponds to the frictional force at a section $\Delta x = 2h\sin\theta_0/2$ (Fig. 1)

$$F_{\ell a0} = 2\mu h \frac{du}{dy} \sin \theta_0/2. \tag{15}$$

At a ribbed surface, the drag force at this section Δx in laminar flow conditions is determined by Eq. (13) with $r_1 = h$

$$F_{\ell a} = 2\mu h \frac{\pi}{\theta_0} \frac{du}{dy} \sin \frac{\theta_0}{2}. \tag{16}$$

Taking account of Eqs. (15) and (16), the additional drag arising at a surface with triangular ribs is obtained

$$\Delta F_{\ell a} = F_{\ell a} - F_{\ell a0} = 2\mu h \left(\frac{\pi}{\theta_0} - 1 \right) \frac{du}{dy} \sin \frac{\theta_0}{2}. \tag{17}$$

Equation (17) shows that, with the given formulation of the problem in laminar flow conditions, the longitudinal triangular ribs lead to increase in drag in comparison with a smooth surface. Studies [6, 7], brought to our attention by Ginevskii, are of interest in this connection. In [6], the possibility of up to 4% reduction in drag in laminar flow conditions along the ribs in comparison with a smooth wall was indicated. In [7], no positive effect of the ribs was found. On the basis of Eq. (17), a relation between $\tau_{\ell a}$ in Eq. (14) and the velocity gradient of the mean flow du/dy may be established. In the region $0 \le y \le h \cos(\theta_0/2)$, a liquid layer bounded above by the coordinate y is considered. A tangential stress $\mu(du/dy)$ and an additional tangential stress due to flow in the corner acts on this layer from the wall. Assuming that the part of the ribbed surface defined by the coordinate $0 \le r \le y$ participates in the creation of the additional drag force (Fig. 1), it follows from Eq. (17) that

$$\Delta F_{\ell a} = 2\mu \left(\frac{\pi}{\theta_0} - 1 \right) y \frac{du}{dy} \sin \frac{\theta_0}{2}.$$

If this force is now referred to the characteristic dimension $2h\sin(\theta_0/2)$, the distance between the rib vertices, the reduced value of the additional tangential stress for a liquid layer with coordinate y is obtained

$$\Delta\tau_{\ell a} = \mu \left(\frac{\pi}{\theta_0} - 1 \right) \frac{y}{h} \frac{du}{dy}.$$

In the region $0 \le y \le h \cos(\theta_0/2)$, the total laminar tangential stress is

$$\tau_{\ell a} = \left[1 + \left(\frac{\pi}{\theta_0} - 1 \right) \frac{y}{h} \right] \mu \frac{du}{dy}. \tag{18}$$

In the region $y > h \cos(\theta_0/2)$, the usual formula is valid

$$\tau_{\ell a} = \mu \frac{du}{dy}. \tag{19}$$

As $\theta_0 \to \pi$, Eq. (18) reduces to Eq. (19). In the limit $\theta_0 \to 0$, Eq. (18) becomes meaningless, since liquid flow in the zone $0 < h \cos(\theta_0/2)$ corresponding to the solid wall cannot be considered.

3. In TBL flow around a solid wall, a viscous (laminar) sublayer is formed in the immediate vicinity of the surface, since the velocity pulsations vanish at the wall. The form of the viscous sublayer at a longitudinal ribbed surface is shown schematically in Fig. 1. The thickness of the laminar sublayer is denoted by δ_0. The zone $0 \le y \le y_*$ is characterized by laminar flow in the corner, where $y_* = \delta_0/\sin(\theta_0/2)$. In the region $y > y_*$, the flow around the rib is turbulent. On the left-hand side of Eq. (14), τ_T is defined as follows: in the region $y_* \le y \le h \cos(\theta_0/2)$

$$\tau_T = \rho l^2 \left(\frac{du}{dy} \right)^2 + \frac{h - y_*}{h} \tau_w; \tag{20}$$

in the region $y > h \cos(\theta_0/2)$

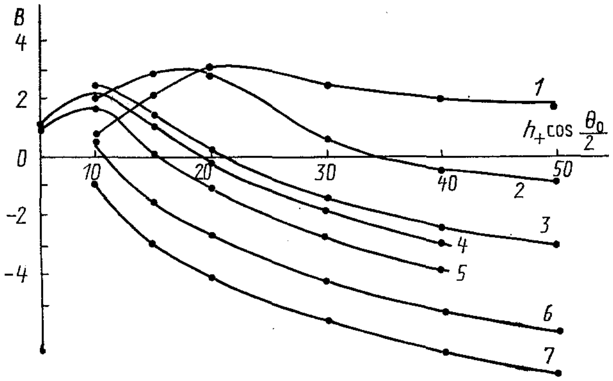

Fig. 2. Dependence of roughness function B on geometric parameters of ribbed surface with $\delta_0 u_*/\nu = 5$: 1) $\theta = 30°$; 2) 45; 3) 60; 4) 64; 5) 72; 6) 90; 7) 110°; $h_+ = hu_*/\nu$.

$$\tau_T = \rho l^2 \left(\frac{du}{dy}\right)^2. \tag{21}$$

The second term on the right-hand side of Eq. (20) approximately determines the additional stress associated with turbulent flow around the upper parts of the longitudinal ribs as a fraction of the total tangential stress acting on the longitudinal rib. The mixing path $l = l(y)$ is usually written in the form $l = \kappa(y - y_0)$, where y_0 is related to the viscous-sublayer thickness [5]. In the case of a ribbed surface with a triangular profile, y_0 may be chosen to be $y_* = \delta_0/\sin(\theta_0/2)$ (Fig. 1), so that

$$l(y) = \varkappa(y - y_*) = \varkappa(y - \delta_0) - \varkappa(y_* - \delta_0) = l_0 - \varkappa\delta_0\left(\frac{1}{\sin\frac{\theta_0}{2}} - 1\right),$$

where $t_0(y)$ is the mixing path at a rough surface. Taking the Van Driest expression for the mixing path $l_0(y)$ [5]

$$l_0(y) = \varkappa y\left[1 - \exp\left(-\frac{yu_*}{26{,}5\nu}\right)\right],$$

it follows that

$$l(y) = \varkappa y\left[1 - \exp\left(-\frac{yu_*}{26{,}5\nu}\right)\right] - \varkappa\delta_0\left(\frac{1}{\sin\frac{\theta_0}{2}} - 1\right), \tag{22}$$

where $\kappa = 0.4$; $u_* = \sqrt{\tau_w/\rho}$; δ_0 is the viscous-sublayer thickness at a smooth wall.

Choosing u_* as the characteristic velocity and ν/u_* as the characteristic linear dimension, the basic Eq. (14) is obtained in the following dimensionless form, taking account of Eqs. (18)-(22): when $0 \le \xi = (yu_*/\nu) \le \delta_0/\sin(\theta_0/2)$ (region I)

$$\left[1 + \left(\frac{\pi}{\theta_0} - 1\right)\frac{\xi}{h}\right]\frac{du}{d\xi} = 1; \tag{23}$$

when $\delta_0/\sin(\theta_0/2) < \xi \le h\cos(\theta_0/2)$ (region II)

$$\frac{du}{d\xi} + \left\{\varkappa\xi\left[1 - \exp\left(-\frac{\xi}{26{,}5}\right)\right] - \varkappa\delta_0\left(\frac{1}{\sin\frac{\theta_0}{2}} - 1\right)\right\}^2\left(\frac{du}{d\xi}\right)^2 - \frac{\delta_0}{h\sin\frac{\theta_0}{2}} = 0; \tag{24}$$

when $\xi > h\cos(\theta_0/2)$ (region III)

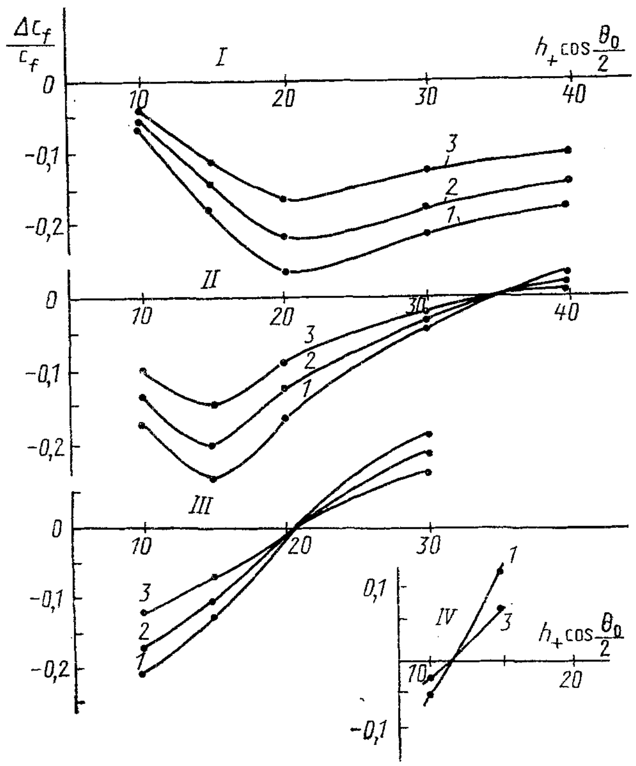

Fig. 3. Dependence of the relative decrease in the local drag coefficient on the geometric parameters of the ribbed surface when $\delta_0 u_*/\nu$ = 5: I) θ_0 = 30°; II) 45; III) 60; IV) 90; 1) $Re_z = 10^6$; 2) 10^7; 3) 10^9.

$$\frac{du}{d\xi}+\left\{\varkappa\xi\left[1-\exp\left(-\frac{\xi}{26,5}\right)\right]-\varkappa\delta_0\left(\frac{1}{\sin\frac{\theta_0}{2}}-1\right)\right\}\left(\frac{du}{d\xi}\right)^2=1. \tag{25}$$

In Eqs. (23)-(25), the previous notation is used for the dimensionless quantities u, δ_0, h.

Integrating Eqs. (23)-(25) with the boundary condition $u(\xi = 0) = 0$ and the matching conditions of $u(\xi)$ at the zone boundaries, the result obtained is:

in region I

$$u(\xi)_{\rm I}=\frac{h}{\frac{\pi}{\theta_0}-1}\ln\left[1+\frac{\xi}{h}\left(\frac{\pi}{\theta_0}-1\right)\right]; \tag{26}$$

in region II

$$u(\xi)_{\rm II}=u_{\rm I}\left(\xi=\delta_0/\sin\frac{\theta_0}{2}\right)+\frac{2\delta_0}{h\sin\frac{\theta_0}{2}}\int\limits_{\delta_0/\sin\frac{\theta_0}{2}}^{\xi}\left\{1+\right.$$

$$\left.+\sqrt{1+\frac{4\delta_0}{h\sin\frac{\theta_0}{2}}\left\{0,4\,\xi\left[1-\exp\left(-\frac{\xi}{26,5}\right)\right]-0,4\,\delta_0\left(\frac{1}{\sin\frac{\theta_0}{2}}-1\right)\right\}^2}\,\right\}^{-1}d\xi; \tag{27}$$

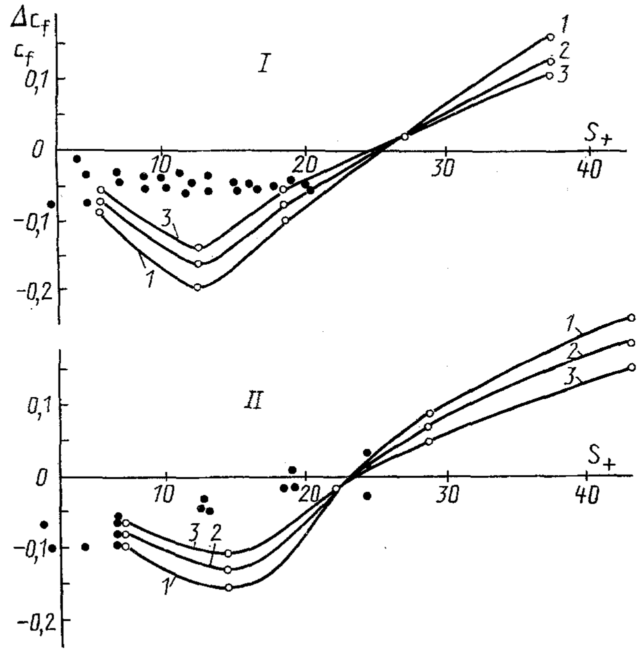

Fig. 4. Comparison of theoretical (curves) and experimental (filled points) values of the relative decrease in local drag coefficient when $\delta_0 u_*/\nu = 5$: I) $\theta_0 = 64°$; II) 72; 1) $Re_z = 10^6$; 2) 10^7; 3) 10^9.

in region III

$$u(\xi)_{III} = u_{II}\left(\xi = h\cos\frac{\theta_0}{2}\right) + 2\int\limits_{h\cos\frac{\theta_0}{2}}^{\xi}\left\{1 + \sqrt{1 + 4\left\{0{,}4\,\xi\left[1 - \exp\left(-\frac{\xi}{26{,}5}\right)\right] - 0{,}4\,\delta_0\left(\frac{1}{\sin\frac{\theta_0}{2}} - 1\right)\right\}^2}\right\}^{-1} d\xi. \quad (28)$$

as $\theta_0 \to \pi$, Eqs. (26)-(28) reduce to the well-known expressions for a rough surface [5].

Calculating the velocity profiles at various values of δ_0, θ_0, h from the given formulas, $B = u(\xi) - u_0(\xi)$ may be determined in the region of logarithmic variation of $u(\xi)$. The dependence $u_0(\xi)$ corresponds to the velocity profile at a smooth plate. Knowing B, the variation in local frictional coefficient Δc_f may be determined from the approximate formula

$$\frac{\Delta c_f}{c_{f_0}} = -B\sqrt{2c_{f_0}}\,, \quad (29)$$

which is valid when $\Delta c_f/c_{f_0} < 1$. Here c_{f_0} corresponds to the velocity distribution $u_0(\xi)$. The results of determining B using Eqs. (26)-(28) when $\delta_0 = 5$ are shown in Fig. 2. Using these data, Eq. (29), and the Schultz-Grunov formulas [8]

$$c_{f_0} = 0{,}370\,(\lg \mathrm{Re}_z)^{-2{,}584}\,,$$

$\Delta c_f/c_{f_0}$ is calculated for various $Re_z = zu_\infty/\nu$. The results are shown in Figs. 3 and 4. For convenience of comparison with experimental data [2], the dimensionless quantity $S_+ = Su_*/\nu$ is plotted along the abscissa in Fig. 4, where S is the distance between vertices of the longitudinal ribs. The theoretical dependences $\Delta c_f/c_{f_0} = f(S_+)$ in Fig. 4 are shown by curves (unfilled points) and the experimental data by filled points. Calculations by the above approximate scheme permit the formulation of the following basic conclusions regarding the influence of the characteristics of triangular longitudinal ribs on the change in drag of a solid surface in a turbulent flow.

1. The decrease in drag depends significantly on the vertex angle of the triangular ribs, other conditions being equal. With decrease in θ_0, the reduction in drag is greater. At $\theta_0 > 90°$, the drag reduction disappears.

2. With fixed θ_0, the decrease in drag depends on the dimensionless quantity $h_+ = hu_*/\nu$ (or S_+). When $h_+ = 0$, the gain in drag $\Delta c_f = 0$ (smooth plate); with increase in h_+, Δc_f increases, reaching a maximum, and then decreases. At a certain value h^*_+, $\Delta c_f = 0$, and then the wall drag increases in comparison with a smooth surface with increase in h_+.

3. Calculations show that, with increase in thickness of the viscous sublayer, the reduction in drag at a ribbed surface is greater; this is confirmed by experiments with polymer solutions.

4. Comparison of the calculation results with the known experimental data indicates satisfactory qualitative agreement, confirming, in our view, the adequacy of the physical model on which the approximate theoretical model proposed here is based.

NOTATION

δ_0, thickness of viscous sublayer; S, distance between the vertices of triangular ribs; h, length of triangular ribs; θ_0, angle between faces of triangular cross section; l, mixing path length; μ, ν, ρ, dynamic and kinematic viscosity, density of liquid; u(y), distribution of mean velocities in boundary layer; $v(r, \theta)$, longitudinal-velocity distribution in the space between the triangular ribs; p, pressure in the space between the ribs; τ_w, tangential frictional stress at wall; κ, Karman constant; c_f, local frictional coefficient at ribbed surface; c_{f_0}, local frictional coefficient at smooth surface; Re_z, local Reynolds number.

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